Infeasible Assignment Problem

Sometimes it is possible a particular person is incapable of performing certain job or a specific job can't be performed on a particular machine. In this case the solution of the problem takes into account of these restrictions so that the infeasible assignment can be avoided.

The infeasible assignment can be avoided by assigning a very high cost to the cells where assignments are restricted or prohibited. This is explained in the following Example.

Example:

A computer centre has five jobs to be done and has five computer machines to perform them. The cost of processing of each job on any machine is shown in the table below.

Computer Machines	Jobs 1	Jobs 2	Jobs 3	Jobs 4	Jobs 5
1	70	30	X	60	30
2	X	70	50	30	30
3	60	X	50	70	60
4	60	70	20	40	X
5	30	30	40	X	70

Because of specific job requirement and machine configurations certain jobs can't be done on certain machines. These have been shown by X in the cost table. The assignment of jobs to the machines must be done on a one to one basis. The objective here is to assign the jobs to the available machines so as to minimize the total cost without violating the restrictions as mentioned above.

Solution

Step 1: The cost Table

Because certain jobs cannot be done on certain machines we assign a high cost say for example 500 to these cells i.e. cells with X and modify the cost table. The revised assignment problem is as follows:

		Jobs 1	2	3	4	5
Computer Machines	1	70	30	500	60	30
	2	500	70	50	30	30
	3	60	500	50	70	60
	4	60	70	20	40	500
	5	30	30	40	500	70

Now we can solve this problem using Hungarian Method as discussed in the previous sections.

Step 2: Find the First Reduced Cost Table

		Jobs 1	2	3	4	5
Computer Machines	1	40	0	470	30	0
	2	470	40	20	0	0
	3	10	450	0	20	10
	4	40	50	0	20	480
	5	0	0	10	470	40

Step 3: Find the Second Reduced Cost Table

Computer Machines		Jobs 1	2	3	4	5
	1	40	0	470	30	0
	2	470	40	20	0	0
	3	10	450	0	20	10
	4	40	50	0	20	480
	5	0	0	10	470	40

Step 4: Determine an Assignment

Computer Machines		Jobs 1	2	3	4	5
	1	40	$\boxed{0}$	470	30	0
	2	470	40	20		0
	3	10	450		$\boxed{20}$	10
	4	40	50	0	20	480
	5	0	~~0~~	$\boxed{10}$	470	40

Step 5:

The solution obtained in Step 4 is not opti~~mal.~~ Because we were able to make four assignments when five were required.

Step 6:

Cover all the zeros of the table shown in the Step 4 with four lines (since already we made four assignments).

Check row 4 since it has no assignment. Note that row 4 has a zero in column 3, therefore check column3. Then we check row 3 since it has a zero in column 3. Note that no other rows and columns are checked. Now we may draw four lines through unchecked rows (row 1, 2, 3 and 5) and the checked column (column 3). This is shown in the table given below

Jobs

Computer Machines	1	2	3	4	5
1	40	$\boxed{0}$	470	30	0
2	470	40	20		0
3	10	450		$\boxed{20}$	10
4	40	50	0	20	480
5		0	$\boxed{10}$	470	40

Step 7:

Develop the new revised table.

Examine those elements that are not cove[redacted] by a line in the table given in Step 6. Take the smallest element in this case the smallest element is 10. Subtract this smallest element from the uncovered cells and add 1 to elements (A6, B6, D6 and F6) that lie at the intersection of two lines. Finally, we get the new revised cost table, which is shown below:

Jobs

Computer Machines	1	2	3	4	5
1	40	0	471	30	0
2	470	40	21	0	0

4	30	40	0	10	470
5	0	0	11	470	40

Step 8:

Now, go to Step 4 and repeat the procedure until we arrive at an optimal solution (assignment).

Step 9:

Determine an assignment

		Jobs				
		1	**2**	**3**	**4**	**5**
	1	40	0	471	30	$\boxed{0}$
Computer Machines	**2**	470	40	21		$\cancel{0}$
	3		440	0	$\boxed{10}$	0
	4	30	40		10	470
	5	$\boxed{0}$		11	470	40

Since the number of assignments equal to the number of rows (columns), the assignment shown in the above tale is optimal.

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Thus, the Machine1 is assigned to Job5, Machine 2 is assigned to job4, Machine3 is assigned to job1, Machine4 is assigned to job3 and Machine5 is assigned to job2.

The minimum assignment cost is: 170